THE FINITE-INTEGRATION BEAM-PROPAGATION METHOD (FIBPM)

Monika Niederhoff, Wolfgang Heinrich, and Peter Russer

Ferdinand-Braun-Institut für Höchstfrequenztechnik Berlin / Germany

ABSTRACT

We present a novel full-vector beam-propagation method where the discretization of Maxwell's equations is performed by finite *integration* (FIBPM). This allows to include thin layers of complex permittivity without increasing mesh density. A FIBPM code for a massively parallel computer has been developed and applied successfully to laser diode structures.

INTRODUCTION

Design and optimization of active and passive optoelectronic devices require simulation of the propagating electromagnetic field. For the analysis of complex waveguide structures, beam-propagation methods (BPM) are well established. They are based on a successive computation of the transverse electromagnetic field distribution along the direction of propagation [1]. The original BPM solves the scalar wave equation using FFT to perform the propagation of the field through the waveguide structure [2]. Recently various full-vector beam-propagation methods based on finite differences have been developed for the analysis of polarization-sensitive structures. The propagation algorithms are based on the method of lines (MOL) [3], finite difference schemes [4, 5, 6] or on Taylor expansion of the matrix-valued exponential function [7]. In any case, the computational effort for the evaluation of the propagating field is considerable. Particularly, waveguide cross-sections containing small details such as quantum wells result in a huge mesh size, and thus way to prohibitive numerical efforts. This problem is addressed by the method presented here.

THEORY

The new basic feature of the FIBPM is that the discretized equations are derived from finite integration, i.e., from integration over elementary cells, rather than from differentiation as done in the common Finite-Difference BPM. This enables one to treat optically thin layers without decreasing mesh size to the thickness' order of magnitude. The approach is described below.

Analytical Derivation of the FIBPM

Discretization is performed by dividing the waveguide structure into rectangular elementary cells of variable size (graded mesh). The thickness of these elementary cells in longitudinal direction is supposed to be infinitesimally small. Within the elementary cells the field components are sampled according to Yee [8]. In order to derive the propagation equation, we start from the integral form of Maxwell's equations in frequency domain postulating a non-magnetic and isotropic medium. By integrating the fields over the elementary cells as has been shown in [9] we obtain equations relating the discretized field components. The remaining part of the derivation follows the well-known beam-propagation approach under paraxial approximation [7]. For propagation, however, we use an algorithm based on the expansion of the matrix exponential into a series of Chebyshev polynomials, which permits arbitrary propagation steps Δz [10, 11, 12], and thus enhances efficiency.

Inclusion of Thin Layers

Reducing the transverse discretization steps, CPU time grows not only because of the increasing matrix size, but also because convergence slows down. For this reason, the performance of finite-difference beam-propagation methods is not satisfactory for waveguide structures containing very thin layers (e. g. quantum wells). Because our propagation method is derived by finite integration, it allows the inclusion of thin layers by effective parameters in a straightforward manner. Hence, the discretization steps need not resolve the layers' thickness, whereby the above-mentioned problem is avoided. The approximation of the integral $\iint \varepsilon \boldsymbol{E} \mathrm{d}A$ across the elementary cell i with the integration area A_i yields $\bar{\varepsilon}_i E_n^i \Delta A^i$, where $\bar{\varepsilon}_i$ is the mean value of ε accross A^i. For the evalution $\oint \boldsymbol{E}(s)\mathrm{d}s$ along an elementary integration path, we make use of the continuity of the normal component of the electric displacement D_n across discontinuities of ε and write

$$\int_{s_i-\Delta s_i/2}^{s_i+\Delta s_i/2} E_s \,\mathrm{d}s = \int_{s_i-\Delta s_i/2}^{s_i+\Delta s_i/2} \tfrac{1}{\varepsilon} D_s \,\mathrm{d}s \approx D_s^i \int_{s_i-\Delta s_i/2}^{s_i+\Delta s_i/2} \tfrac{1}{\varepsilon} \,\mathrm{d}s = E_s^i \varepsilon_i \int_{s_i-\Delta s_i/2}^{s_i+\Delta s_i/2} \tfrac{1}{\varepsilon} \,\mathrm{d}s = E_s^i \, \overline{\Delta s_i} \,, \quad (1)$$

where $\overline{\Delta s_i}$ is either $\overline{\Delta x_i}$ or $\overline{\Delta y_i}$.

We thus obtain the effective parameters $\bar{\varepsilon}_i$, $\overline{\Delta x_i}$ und $\overline{\Delta y_i}$, which allow us to treat discontinuous variations of the permittivity ε *within* the elementary cells.

Massive Parallelization

The code of the FIBPM is designed for the massively parallel computer DECmpp12000 with an array of 4096 processors. By mapping the discretization mesh on the processor array each elementary cell is associated with an elementary processor. The evaluation of the expansion terms is performed simultaneously for all elementary cells (Single Instruction Multiple Data: SIMD). Because only neighbouring field components are interconnected, the exchange of data is limited to the communication between neighbouring processors. Thus the massively parallel architecture is used efficiently.

RESULTS

In order to check the new method and to demonstrate its benefits in treating thin layers, we calculated the propagation constants of the planar waveguide in Fig. 1 for various values of d_2. Table 1 compares results of the FIBPM to results of a simplified method where the thin layer is included by the mean value of ε in the elementary cell. Table 1 shows that by using the thin-layer approximation, as described in the above section, the error of the FIBPM is reduced to the discretization error. With decreasing d_2 the advantage of the FIBPM over the mean value method increases

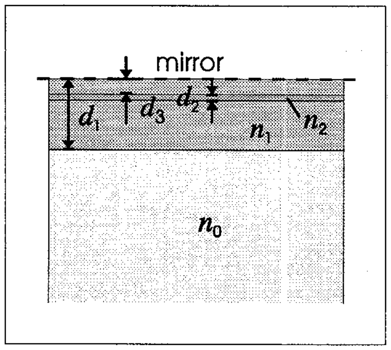

Fig. 1: Structure of planar waveguide that was used to obtain the results Tab. 1

Method:	analytical	FIBPM		mean value	
	n_{eff}	n_{eff}	$\frac{\Delta n_{\mathrm{eff}}}{n_{\mathrm{eff}}}$	n_{eff}	$\frac{\Delta n_{\mathrm{eff}}}{n_{\mathrm{eff}}}$
Structure 1:					
TE	1.05841	1.05825	$3.8 \cdot 10^{-5}$	same as FIBPM	
TM	1.05439	1.05467	$6.6 \cdot 10^{-4}$	same as FIBPM	
Structure 2:					
TE	1.06217	1.06204	$1.2 \cdot 10^{-4}$	1.06198	$1.8 \cdot 10^{-4}$
TM	1.05763	1.05787	$2.2 \cdot 10^{-4}$	1.05802	$3.7 \cdot 10^{-4}$
Structure 3:					
TE	1.06221	1.06217	$3.8 \cdot 10^{-5}$	1.06198	$2.2 \cdot 10^{-4}$
TM	1.05743	1.05762	$1.8 \cdot 10^{-4}$	1.05802	$5.6 \cdot 10^{-4}$

Tab. 1: Inclusion of thin layers: Comparison of FIBPM with mean value approximation; Effective index and normalized propagation constant of TE and TM fundamental modes of the planar waveguide in Fig. 1; Discretization $\Delta x = 0.1\lambda$; $n_0 = 1.0$, $n_1 = \sqrt{1.2}$, $d_1 = 0.50\lambda$, $d_3 = 0.12\lambda$; Structure 1: $d_2 = 0$; Structure 2: $n_2 = \sqrt{1.3}$, $d_2 = 0.04\lambda$; Structure 3: $n_2 = \sqrt{1.4}$, $d_2 = 0.02\lambda$.

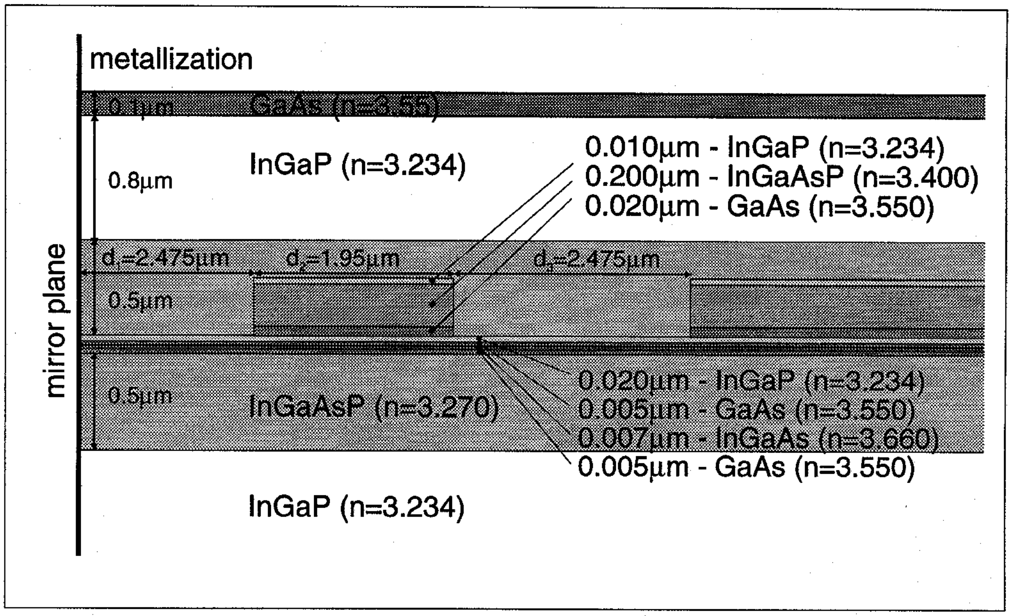

Fig. 3: Lateral ARROW structure on which the field calculation in Fig. 4 is based

further.

To examine the coupling between orthogonal field components we simulated a buried waveguide. Figure 2 shows the field distribution of the fundamental mode. To check the FIBPM for a more complex waveguide structure we simulated a lateral ARROW (Anti Resonant Reflecting Optical Waveguide) structure. High-power laser diodes of this type are developed at the Ferdinand-Braun-Institut. For details of the structure, see Fig. 3. The cross-section was discretized by 64×128 cells, where $\Delta x = 88.1\text{nm}$ and $\Delta y = 75.0\text{nm}$. The propagation step was $\Delta z = 4\mu\text{m}$. Figure 4 shows the near field of the TE fundamental mode.

CONCLUSIONS

A full-vector beam-propagation method based on finite integration is presented. The method is especially suited for structures containing very thin layers such as quantum wells. Chebyshev expansion of the matrix exponential permits arbitrary propagation steps Δz, thus increasing the numerical efficiency of the propagation algorithm considerably. A code for running the simulation on a massively parallel computer was developed. Results for typical structures are presented. The method proves to be a versatile tool for the efficient simulation of optical waveguide structures with cross-sections of complex geometry.

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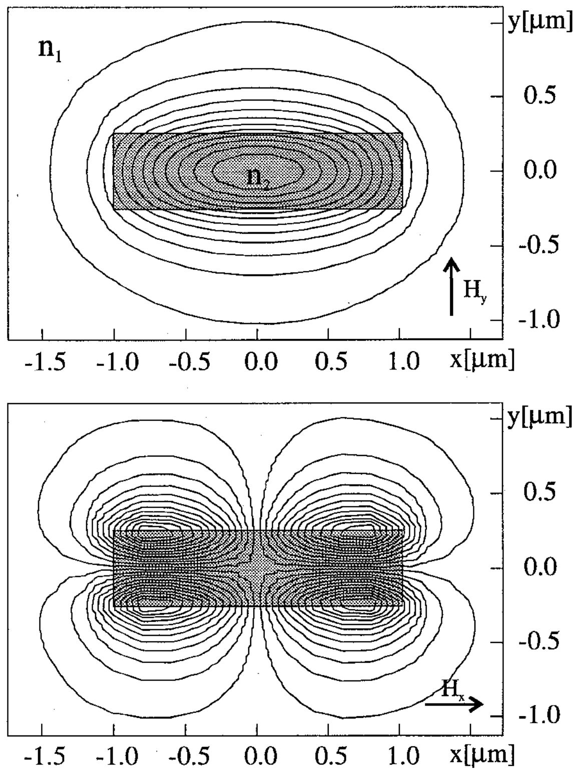

Fig. 2: Vector field distribution of the fundamental mode of a buried waveguide; $n_1 = 3.150675$, $n_2 = 3.398916$, $\lambda = 1.55\mu$m, $\Delta x = 200$nm, $\Delta y = 62.5$nm, $n_{\text{eff}} = 3.27768$

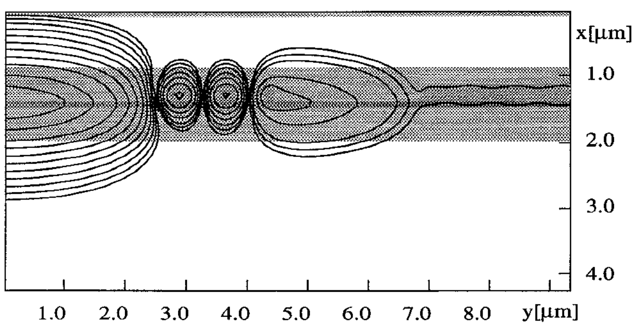

Fig. 4: Transversal near-field distribution of the TE fundamental mode of the lateral ARROW structure in Fig. 3, $\lambda = 940$nm, $n_{\text{eff}} = 3.26343$

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